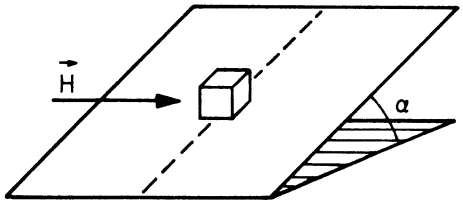
rough inclined plane

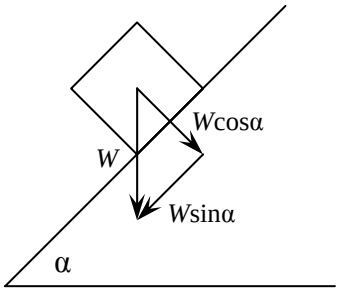


A particle of weight W rests on a rough inclined plane that makes an angle alpha with the horizontal.

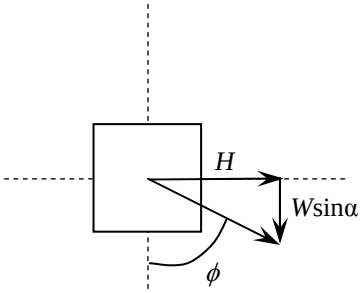
(a) If the co-efficient of static friction $\mu = 2\tan\alpha$, find the least horizontal force H_{min}, acting transverse to the slope of the plane that will cause the particle to move.
(b) In what direction will it go?

Solution by Rudy Arthur:

free body diagram



top view



The force on the block in the plane is $\vec{H} + \vec{W}\sin\alpha$, while the minimum force required to overcome static friction is $(\vec{W}\cos\alpha)(2\tan\alpha)$. We can find H_{min}, the force that will just cause the particle to move, by equating these and squaring:

$$(\vec{H}_{min} + \vec{W}\sin\alpha)^2 = ({H_{min}}^2 + \vec{H}_{min}\cdot\vec{W}\sin\alpha + W^2\sin^2\alpha) = ((\vec{W}\cos\alpha)(2\tan\alpha))^2 .$$

After some cancellation (recalling that $\vec{H}$ and $\vec{W}$ are perpendicular, so their dot product is zero) this becomes,

(a) $H_{min} = \sqrt{3}W\sin\alpha$.

When $H = H_{min}$ the components of the force in the inclined plane are $\sqrt{3}W\sin\alpha$ (horizontal) and $W\sin\alpha$ (vertical), so $\tan\phi = \sqrt{3}$. Thus,

(b) the particle moves down at an angle $\phi = 60^\circ$.