pion, muon, neutrino

A pion ($m_\pi = 273\ m_e$) at rest decays into a muon ($m_\mu = 207\ m_e$) and a neutrino ($m_\nu = 0$). Find the kinetic energy and momentum of the muon and the neutrino in MeV.

Solution by Michael A. Gottlieb:

(I choose units such that $c = 1$, and assume that $m_e = 0.511$MeV.)

Since the pion is at rest conservation of momentum dictates that the momenta of the muon and the neutrino be equal in magnitude (and opposite in direction),

$$p_\mu = p_\nu \,. \qquad \text{(A)}$$

Since the pion is at rest its energy equals its mass, $E_\pi = m_\pi$. Since the neutrino is massless its energy equals its momentum, $E_\nu = p_\nu$. By conservation of energy, $E_\pi = E_\mu + E_\nu$, so

$$E_\mu = m_\pi - p_\nu \,. \qquad \text{(B)}$$

Substituting the right sides of (A) and (B) into the left side of the fundamental kinematic equation for the muon $E_\mu^2 - p_\mu^2 = m_\mu^2$ yields

$$\left(m_\pi - p_\nu\right)^2 - p_\nu^2 = m_\mu^2 \,.$$

Solving for p_ν gives (the magnitudes of) the momenta of the decay particles and the kinetic energy (equal to the total energy) of the massless neutrino,

$$p_\nu \left(= p_\mu = E_\nu\right) = \frac{\left(m_\pi^2 - m_\mu^2\right)}{2m_\pi} = 29.65 \text{ MeV} \,.$$

The kinetic energy of the muon equals its total energy minus its mass which, using (B), is $\left(m_\pi - p_\nu\right) - m_\mu =$ 4.08 MeV .