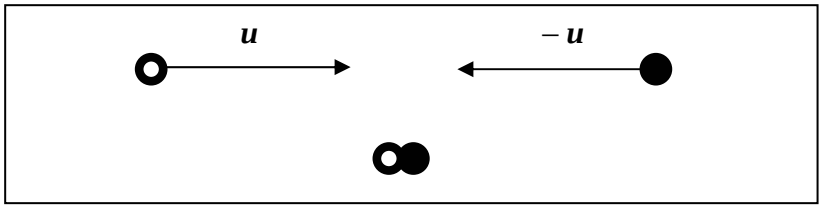
Inelastic Relativistic Collision

A particle of mass m, moving at speed $v = 4c/5$, collides inelastically with a similar particle at rest.

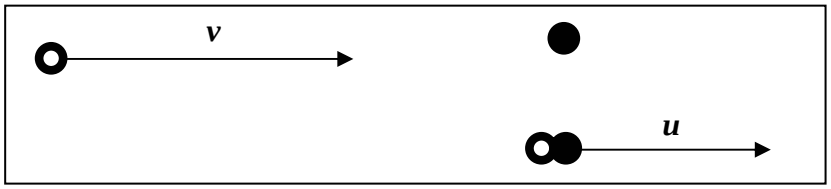
(a) What is the speed v_C of the composite particle?
(b) What is its mass m_C?

Solution by Ilkka Mäkinen:

Call the frame of the particle at rest "the lab frame" and consider the center-of-mass (CM) frame.



CM frame



lab frame

In order for momentum to be conserved the center-of-mass of the system must maintain a constant velocity $\boldsymbol{u}$; this will be the velocity of the composite particle in the lab frame.

The particle moving at speed v in the lab frame moves at speed u in the CM frame, while $-u$ is the speed of the lab frame relative to the CM frame. We can thus use the relativistic transformation of velocities to find u:

$$\frac{v-u}{1-vu} = u \quad \rightarrow \quad vu^2 - 2u + v = 0$$

$$u = \frac{1}{v} - \sqrt{\frac{1}{v^2} - 1} = \frac{5}{4} - \sqrt{\frac{9}{16}} = \frac{1}{2}$$

Then we can find the composite particle's mass m_C from the conservation of momentum:

$$\frac{mv}{\sqrt{1-v^2}} = \frac{m_C u}{\sqrt{1-u^2}}$$

$$\therefore m_C = \frac{v}{u}\frac{\sqrt{1-u^2}}{\sqrt{1-v^2}}m = \frac{8}{5}\frac{\sqrt{3}}{2}\frac{5}{3}m = \frac{4}{\sqrt{3}}m\,.$$