Inelastic Relativistic Collision

A particle of mass m, moving at speed $v = 4c/5$, collides inelastically with a similar particle at rest.

(a) What is the speed v_C of the composite particle?

(b) What is its mass m_C?

Solution by Michael Gottlieb:

(*I choose units for which* $c = 1$.)

Call the moving particle 'M', and the particle at rest 'R'. (The composite particle is defined to be 'C'.)

The momentum and energy of the particle at rest are

$$p_R = 0 \quad E_R = m .$$

The momentum of the moving particle is

$$p_M = \frac{mv}{\sqrt{1-v^2}} = \frac{4/5}{\sqrt{1-\left(\frac{4}{5}\right)^2}} m = \frac{4}{3} m ,$$

and its energy is

$$E_M = \frac{p_M}{v} = \frac{(4/3)m}{4/5} = \frac{5}{3} m ,$$

For the composite particle, the conservation of energy implies that

$$E_C = E_M + E_R = \frac{8}{3} m ,$$

while the conservation of momentum implies that

$$p_C = p_M = \frac{4}{3} m .$$

The speed of the composite particle is

$$v_c = \frac{p_c}{E_c} = \frac{(4/3)m}{(8/3)m} = \frac{1}{2} . \quad (\text{For } c \neq 1,\ v_C = \frac{c}{2} .)$$

The mass of the composite particle is given by the (positive) solution to

$$m_C^2 = E_C^2 - p_C^2 = \left(\frac{8}{3} m\right)^2 - \left(\frac{4}{3} m\right)^2 ,$$

$$m_C = \frac{4}{\sqrt{3}} m .$$