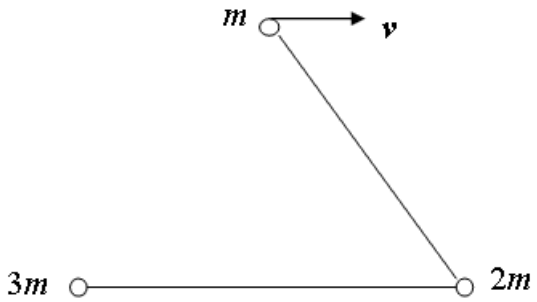
three balls



Three small balls of masses *m*, 2*m* and 3*m* are placed on a smooth horizontal surface so that they lie on the vertices of an equilateral triangle. The masses *m* and 2*m*, as well as 2*m* and 3*m*, are connected by light inextensible strings. Initially the strings are taut and the mass *m* is given a velocity ***v*** on the surface in a direction parallel to the string connecting 3*m* and 2*m*. With what velocity does mass 3*m* eventually starts moving? (Neglect friction and the mass of the strings.)

Solution by David Peterson:

Elastic collisions occur between the balls when the strings connecting them become taut: first between *m* and 2*m*, then between 2*m* and 3*m*. For an elastic collision in one dimension[†] between masses m_1 and m_2 having initial velocities u_1 and u_2, and final velocities v_1 and v_2,

$$v_1 = \frac{u_1(m_1 - m_2) + 2m_2u_2}{m_1 + m_2}, \quad v_2 = \frac{u_2(m_2 - m_1) + 2m_1u_1}{m_1 + m_2}. \tag{1}$$

Noting that each collision occurs between a moving ball and one at rest, along a line 60° from the velocity of the moving ball, we solve (1) for the final velocity of 2*m* after the first collision, by setting $m_1 = m$, $u_1 = v\cos 60^\circ$, and $m_2 = 2m$, $u_2 = 0$,

$$v_2 = \frac{2m(v/2)}{3m} = \frac{v}{3}. \tag{2}$$

Next we solve (1) for the final velocity of 3*m* after the second collision, setting $m_1 = 2m$, $u_1 = \frac{v}{3}\cos 60^\circ$, and $m_2 = 3m$, $u_2 = 0$,

$$v_2 = \frac{2(2m)(v/6)}{5m} = \frac{2v}{15}. \tag{3}$$

[†]See http://en.wikipedia.org/wiki/Elastic_collision#One-dimensional_Newtonian .