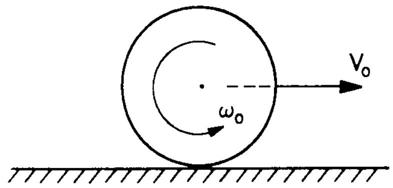
Shooting Marbles



An amusing trick is to press a finger down on a marble, on a horizontal table top, in such a way that the marble is projected along the table with an initial linear speed V0 and an initial backward rotational speed ω0, ω0 being about a horizontal axis perpendicular to V0. The coefficient of sliding friction between marble and top is constant. The marble has radius R.

a) What relationship must hold between Vo, R, and ω0 for the marble to slide to a complete stop?

b) What relationship must hold between Vo, R, and ω0 for the marble to skid to a stop and then start returning toward its initial position, with a final constant linear speed of 3/7 Vo?

Sukumar Chandra's Solution *(using conservation of angular momentum)*

a) When the marble slides to a complete stop, its angular momentum is zero. As the external force, force of kinetic friction, acts horizontally through the point of contact so angular momentum about the starting point is conserved. Initial angular momentum, about the starting point, by virtue of translation of centre of mass is MV_0R clockwise and by virtue of rotation of all the particles of marble about c.m. is $I_{cm}\omega_0$ anti clockwise where $I_{cm} = 2MR^2/5$, is the moment of inertia of the spherical marble about its c.m. Hence initial total angular momentum is $I_{cm}\omega_0 - MV_0R$. As angular momentum is conserved so $I_{cm}\omega_0 - MV_0R = 0$, or, $V_0 = 2\omega_0R/5$.

b) As the final linear speed is constant so there is no sliding *i.e.* the marble is executing pure rolling motion. So its constant angular speed now is anti clockwise ω such that $\omega R = 3V_0/7$. So total angular momentum now is $I_{cm}\omega + 3MV_0R/7$ (both the components being anti clockwise). Applying the principle of conservation of angular momentum we get $I_{cm}\omega_0 - MV_0R = I_{cm}\omega + 3MV_0R/7$, or, $V_0 = \omega_0R/4$