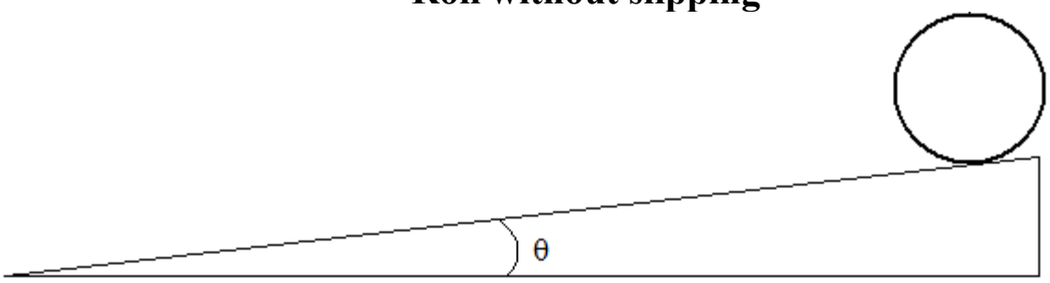
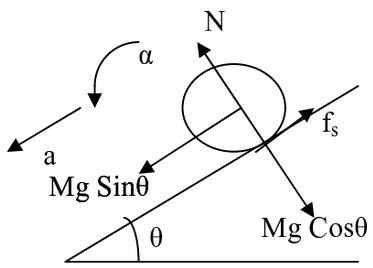
Roll without slipping



A uniform solid ball is placed at rest on an incline of slope angle. What is the minimum value μ_0 of the coefficient of static friction between ball and incline so that the ball will roll down the incline without slipping?

Solution by Sukumar Chandra:



Let...

mass of the solid ball = M, its radius = R,

translational acceleration of centre of the ball down the incline = a,

angular acceleration of the ball about its centre = $\alpha = a/R$, as it is rolling,

$I = 2MR^2/5$ = Moment of inertia of the ball about its centre.

Forces on the ball are...

1. component of its weight down the incline, MgSinθ.
2. component of its weight normal to the incline and into it, MgCosθ.
3. normal reaction force from incline surface, normal to the incline and out of it, N.
4. As the ball is rolling so its point of contact with the incline is stationary but it has a tendency to slip due to the force MgSinθ. Hence static force of friction comes into play and acts on the ball through the point of contact along the incline in upward direction, f_s.

Equation of motion along the incline:

$$MgSin\theta - f_s = Ma \qquad (1)$$

Equation of motion normal to the incline:

$$N - MgCos\theta = 0 \qquad (2)$$

Equation of rotational motion about the centre of the ball:

$$f_s R = I\,\alpha,$$

.

Thus (as $a = R\alpha$)

$$f_s = 2Ma/5 \qquad (3)$$

Eliminating a from equations (1) and (3), we get

$$f_s = 2MgSin\theta / 7, \qquad (4)$$

while equation (2) gives

$$N = MgCos\theta. \qquad (5)$$

But $f_s \leq \mu_0 N$, so $2MgSin\theta / 7 \leq \mu_0 MgCos\theta$, or

$$\mu_0 \geq 2Tan\theta / 7. \qquad (6)$$

Hence the minimum value of μ_0 is $\frac{2}{7}Tan\theta$.