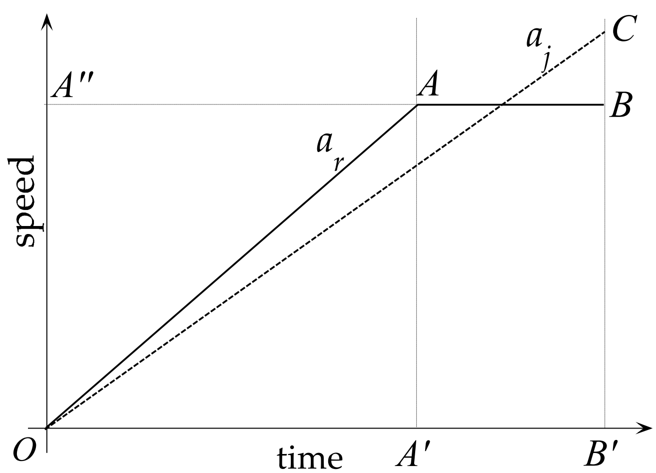
rocket vs. jet

On the long horizontal test track at Edwards AFB, both rocket and jet motors can be tested. On a certain day, a rocket motor, started from rest, accelerated constantly until its fuel was exhausted, after which it ran at constant speed. It was observed that this exhaustion of fuel took place as the rocket passed the midpoint of the measured test distance. The jet motor was started from rest down the track, with a constant acceleration for the entire distance. It was observed that both rocket and jet motors covered the test distance in exactly the same time. What was the ratio of acceleration of the jet motor of that of the rocket motor?

Solution by Riccardo Borghi

We want to propose a purely geometrical derivation of the solution of this problem. To this aim, the figure below shows the temporal diagram of the speeds of the rocket (solid curve) and of the jet (dashed line) during all phases of motion. The length of the segment *OB'* represents the total time (which is identical for the rocket and for the jet) needed to cover the measured test distance. Moreover, the 'slopes' of the segments *OA* and *OC* give the accelerations of the rocket a_r (until the fuel exhaustion) and of the jet a_j, respectively.



The rocket exhausts its fuel after covered half of the test distance: this implies that the areas of the triangle *OAA'* and of the rectangle *A'ABB'* must be identical and, by symmetry, also equal to the area of the triangle *OA''A*. As a consequence, the area of the rectangle *OA''BB'* turns out to be 3/2 of the area of the rectangle *OA''AA'* . Since the two rectangles have a common height (the segment *BB'*), it follows at once that the time of fuel exhaustion (corresponding to the length of *OA'*) is 2/3 of the total time (length of *OB'*). Finally, on imposing that the area of the triangle *OCB'* (the test distance) is twice the area of the triangle *OAA'* (the midpoint of the test distance), we have

$$\frac{1}{2}\, a_j\, \overline{OB'}^2 \,=\, 2 \times \left(\frac{1}{2}\, a_r\, \overline{OA'}^2\right) \Longrightarrow \frac{a_r}{a_j} \,=\, \frac{1}{2} \times \left(\frac{3}{2}\right)^2 \,=\, \frac{9}{8}\,.$$