rocket vs. jet

On the long horizontal test track at Edwards AFB, both rocket and jet motors can be tested. On a certain day, a rocket motor, started from rest, accelerated constantly until its fuel was exhausted, after which it ran at constant speed. It was observed that this exhaustion of fuel took place as the rocket passed the midpoint of the measured test distance. The jet motor was started from rest down the track, with a constant acceleration for the entire distance. It was observed that both rocket and jet motors covered the test distance in exactly the same time. What was the ratio of acceleration of the jet motor of that of the rocket motor?

Solution by Michael A. Gottlieb:

Let: t_R = period of time the rocket accelerated (to the midpoint of the track),
t_C = period of time the rocket coasted (to the end of the track),
t_J = period of time the jet accelerated (to the end of the track).
(It is given that $t_J = t_R + t_C$.)

The rocket covered an equal distance when it is accelerated and when it coasted,

$$\tfrac{1}{2}a_R t_R^2 = \left(a_R t_R\right) t_C ,$$

so $t_C = \tfrac{1}{2}t_R$.

The distance the rocket covered when it accelerated equals half the distance the jet covered when it accelerated,

$$\frac{1}{2}a_R t_R^2 = \frac{1}{2}\left(\frac{1}{2}a_J t_J^2\right) = \frac{1}{4}a_J\left(t_R + t_C\right)^2 = \frac{1}{4}a_J\left(t_R + \tfrac{1}{2}t_R\right)^2 .$$

So,

$$\frac{a_J}{a_R} = \frac{2t_R^2}{\left(t_R + \tfrac{1}{2}t_R\right)^2} = \frac{2}{\left(3/2\right)^2} = \frac{8}{9} .$$