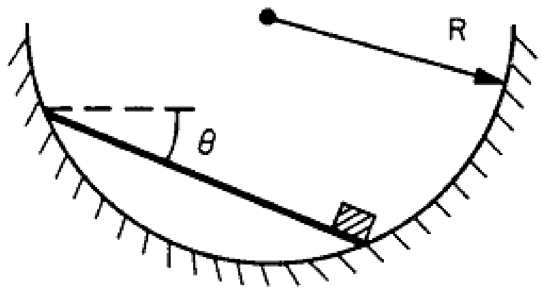
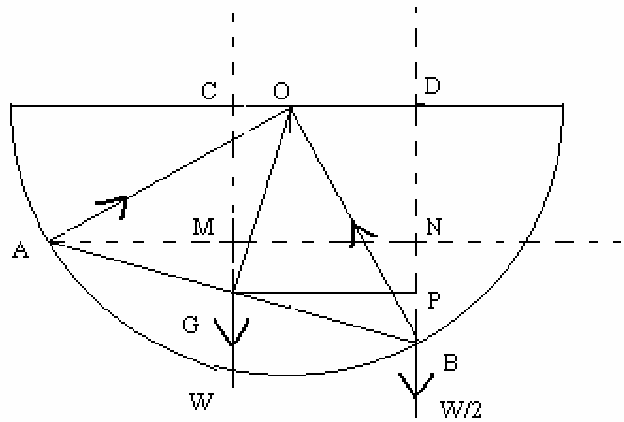
Plank weight trough



A plank of weight W and length $\sqrt{3}R$ lies in a smooth circular trough of radius. At one end of the plank is a weight W/2. Calculate the angle θ at which the plank lies when it is in equilibrium

<u>**Solution by Sukumar Chandra**</u> *(using potential energy)*



$\angle NAB = \angle PGB = \angle CGO = \theta$

We know that when any object is in equilibrium its potential energy is an extremum. Also from calculus we know that wherever a function has an extremum its first derivative is zero. Choosing the diameter of the bowl as the datum level of potential energy, we can express the potential energy of the rod particle system as

$$\text{P.E.} = -\left(W \times CG + \frac{W}{2} \times DB\right) = -W(CG + DB/2)\,. \qquad \text{(I)}$$

(The potential energy is negative because the masses are below datum level.)

G is the midpoint of the chord AB, so OG is perpendicular to AB, and in right-angled triangle AGO,

$$OG^2 = OA^2 - AG^2 = R^2 - (\sqrt{3}R/2)^2 = R^2/4,$$

or OG = R/2. Thus, in right-angled triangles OCG and PGB,

$$CO = OG\, Sin\theta = (R\, Sin\theta)/2\ ,$$

$$PB = GB\, Sin\theta = (\sqrt{3}R\ Sin\theta)/2.$$

So,

$$DB = DP + PB = CG + PB = (R/2)\,(Cos\theta + \sqrt{3}Sin\theta).$$

Putting the values of CG and DB into equation (I) we get

$$P.E. = -\,W(R/2)\,(Cos\theta + (Cos\theta + \sqrt{3}Sin\theta)/2\} = -\,(WR/4)\,(3Cos\theta + \sqrt{3}Sin\theta)$$

The first derivative of P.E. with respect to θ is

$$dP.E./d\theta = -\,(WR/4)\,(-3Sin\theta + \sqrt{3}Cos\theta).$$

Equating this to zero we get

$$-3Sin\theta + \sqrt{3}Cos\theta = 0,$$

or

$$Tan\theta = 1/\sqrt{3}.$$

Hence, $\theta = 30^0$.