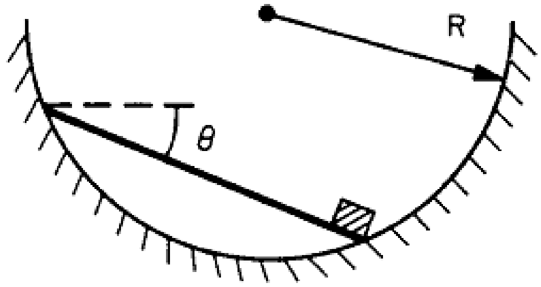
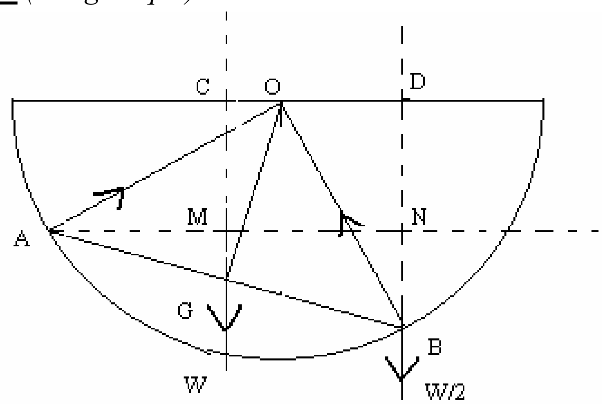
# Plank weight trough




A plank of weight W and length $\sqrt{3}R$ lies in a smooth circular trough of radius. At one end of the plank is a weight W/2. Calculate the angle θ at which the plank lies when it is in equilibrium

**Solution by Sukumar Chandra** *(using torque)*




$$\angle NAB = \angle OGC = \theta$$

The rod AB is in equilibrium under the action of four forces: two normal reactions at A and B exerted by the bowl pass through the centre of the bowl O, weight W of the rod passing through centre of the rod G and vertically downward, whose line of action meets the diameter of the bowl at C, and weight W/2 of the block vertically downward at B, whose line of action meets the diameter of the bowl at D. (As the bowl is smooth there can be no tangential reaction forces at A and B.) Rotational equilibrium of the rod demands that resultant torque of all the forces about any point is zero. Taking the torque of all the forces about point O, we find that the two normal reactions, as they pass through O, do not produce any torque. Hence torque produced by W about O must be balanced by counter-torque produced by W/2 about O. Mathematically this can be expressed as

$$W \times CO = \frac{W}{2} \times DO \quad . \qquad \text{(I)}$$

G is the midpoint of the chord AB, so OG is perpendicular to AB, and in right-angled triangle AGO,

$$OG^2 = OA^2 - AG^2 = R^2 - (\sqrt{3}R/2)^2 = R^2/4,$$

or OG = R/2. So, in right-angled triangles OCG and AMG,

$$CO = OG\, Sin\theta = (R Sin\theta)/2\ ,$$

$$AM = AG\, Cos\theta = (\sqrt{3}R\ Cos\theta)/2.$$

But CD = MN = AM (M being the midpoint of AN). Thus,

$$DO = CD - CO = (\sqrt{3}R\ Cos\theta)/2 - (R Sin\theta)/2 = \frac{R}{2}(\sqrt{3}Cos\theta - Sin\theta).$$

Putting these values of CO and DO into equation (I), we get

$$W \times (R Sin\theta)/2 = \frac{W}{2} \times \frac{R}{2}(\sqrt{3}Cos\theta - Sin\theta)$$

$$\Rightarrow \quad Sin\theta = \frac{1}{2}(\sqrt{3}Cos\theta - Sin\theta) \Rightarrow 2\ Sin\theta = \sqrt{3}Cos\theta - Sin\theta$$

$$\Rightarrow \quad 3Sin\theta = \sqrt{3}Cos\theta \ \Rightarrow \ Tan\theta = \frac{1}{\sqrt{3}}\ .$$

Hence, $\theta = 30^0$.