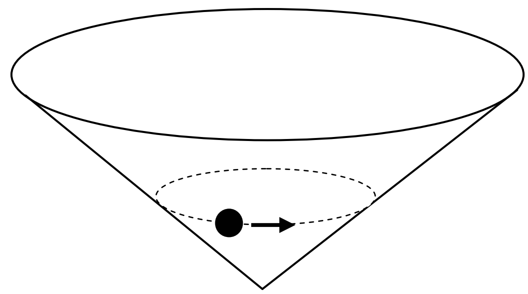
Particle in Cone

by Michael A. Gottlieb



A particle slides at constant speed in a horizontal circular path on the frictionless inner surface of an inverted right circular cone. How is the particle's kinetic energy related to its potential energy?

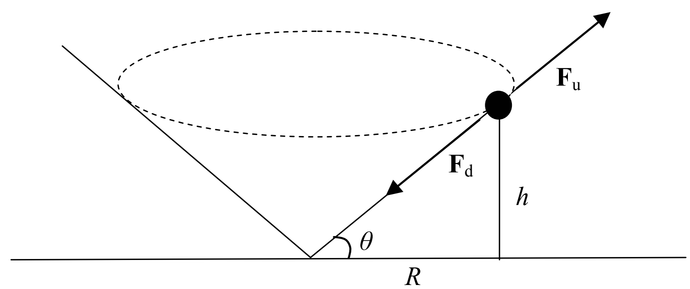


Figure 1

Michael A. Gottlieb's Solution: Assume the cone meets the horizontal at angle θ, and that the particle is circling at height h and lateral distance R from the apex of the cone, such that $\tan\theta = h/R$. For the particle to remain at height h the net force pulling it down toward the apex $\mathbf{F_d}$ must equal the net force pulling it up away from the apex $\mathbf{F_u}$. (Figure 1.)

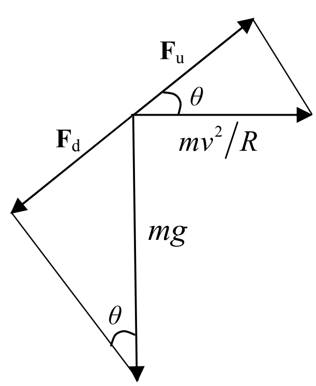


Figure 2

The forces on the particle are from gravity mg down, and from centrifigul acceleration mv^2/R horizontally away from the apex, where v is the speed of the particle. Resolving forces $\mathbf{F}_d$ and $\mathbf{F}_u$ we find that $\mathbf{F}_d = mg\sin\theta$ and $\mathbf{F}_d = \left(mv^2/R\right)\cos\theta$. Setting $\mathbf{F}_d = \mathbf{F}_u$ and simplifying yields $\tan\theta = v^2/gR$.

Recalling that $\tan\theta = h/R$, we find that $v^2 = gh$. Taking the apex of the cone as the datum level for potential energy, we conclude that the particle's kinetic energy ($\frac{1}{2}mv^2$) is equal to half its potential energy (mgh).