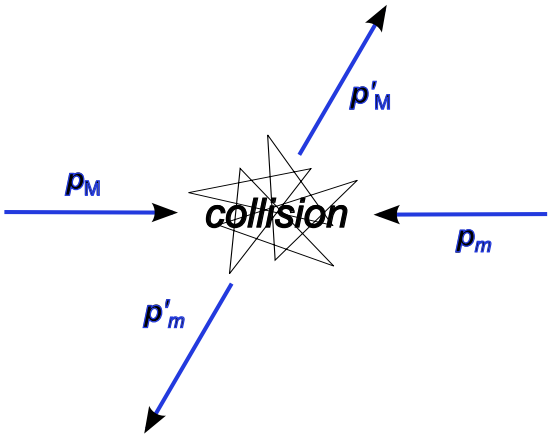
maximum angle of deflection

A moving particle of mass M collides perfectly elastically with a stationary particle of mass $m < M$. Find the maximum possible angle through which the incident particle can be deflected.

Solution by Manuel Fortin (edited by Michael A. Gottlieb)

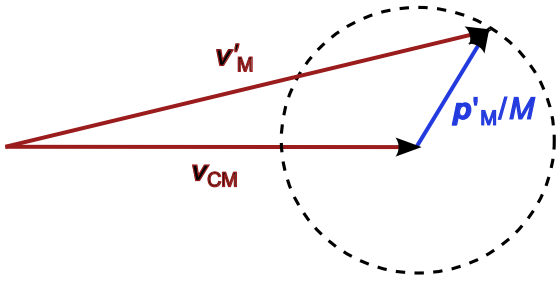
In **the center-of-mass frame** the momenta of the particles are opposite in direction and equal in magnitude both before and after the collision, and conservation of energy dictates that their magnitudes do not change.



Elastic two-body collision in CM frame

$$|\boldsymbol{p}_{\mathrm{M}}| = |\boldsymbol{p}_{\mathrm{m}}| = |\boldsymbol{p}'_{\mathrm{m}}| = |\boldsymbol{p}'_{\mathrm{M}}|$$

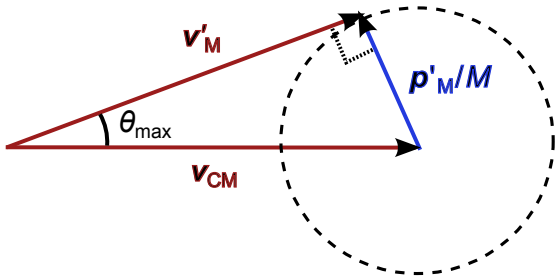
In **the lab frame** the velocity of the incident particle is the sum of the velocity of the center-of-mass and that of the particle in the center-of-mass frame. After the collision the latter has magnitude $|\boldsymbol{p}'_{\mathrm{M}}|/M$ and can be in any direction.



After the collision

$$\boldsymbol{v}'_{\mathrm{M}} = \boldsymbol{v}_{\mathrm{CM}} + \boldsymbol{p}'_{\mathrm{M}}/M$$

The velocity of the incident particle before the collision is parallel to that of the center-of-mass; thus the maximum angle of deflection equals the maximum angle between the velocity of the center-of-mass and that of the incident particle after the collision, as shown below.



Maximal angle of deflection

Therefore,

$$\sin\theta_{\max} = \left(p'_{\mathrm{M}}/M\right)/v_{\mathrm{CM}}\,.$$

Since

$$v_{\mathrm{CM}} = \left(Mv_{\mathrm{M}} + mv_{\mathrm{m}}\right)/\left(M+m\right) = Mv_{\mathrm{M}}/\left(M+m\right),$$

and

$$p'_{\mathrm{M}}/M = p_{\mathrm{M}}/M = v_{\mathrm{M}} - v_{\mathrm{CM}} = mv_{\mathrm{M}}/\left(M+m\right),$$

we conclude that

$$\sin\theta_{\max} = m/M\,.$$