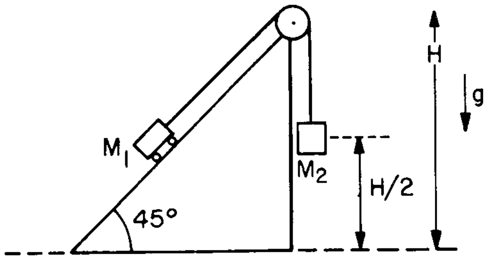
mass on an incline



A mass M_1 slides on a 45º inclined plane of height H as shown. It is connected by a flexible cord of negligible mass over a small pulley (neglect its mass) to an equal mass M_2 hanging vertically as shown. The length of the cord is such that the masses can be held at rest both at height H/2. The dimensions of the masses and the pulley are negligible compared to H. At time t = 0 the two masses are released.

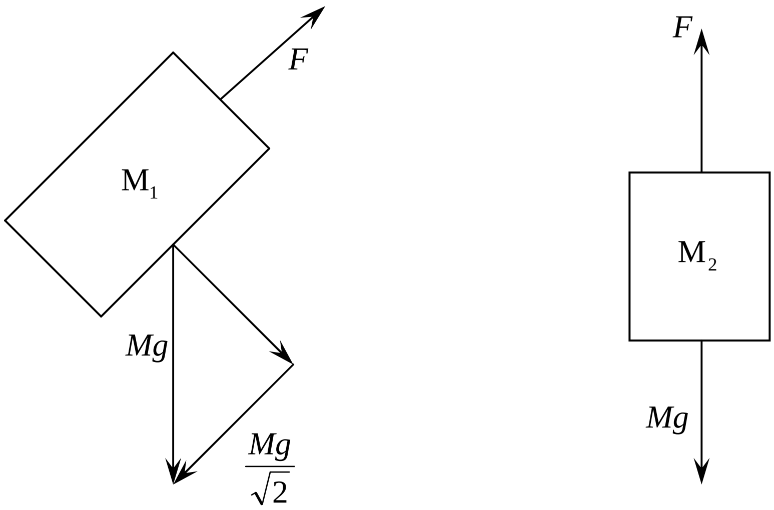
(a) For $t > 0$ calculate the vertical acceleration of M_2

(b) Which mass will move downward? At what time t_1 will it strike the ground

(c) If the mass in (b) stops when it hits the ground, but the other mass keeps moving, show whether or not it will strike the pulley.

Solution by Rudy Arthur:

Free body diagrams:



Let F be the tension in the cord, and let a be the acceleration of the masses.

(a) Newton's second law tells us that

for M_1: $$F - \frac{Mg}{\sqrt{2}} = Ma\,,$$

for M_2: $$F - Mg = -Ma\,.$$

Solving these equations for a , we find the vertical acceleration of M_2

$$a = \frac{1}{2}(1 - \frac{1}{\sqrt{2}})g \quad .$$

(b) M_2 moves downward because the net (downward) force on M_2, which equals Mg , is greater than the net (inclined) force on M_1, which equals $\frac{Mg}{\sqrt{2}}$. We use the the kinematic formula $s = v_0 t + \frac{1}{2}at^2$ to find the time t_1 when M_2 hits the ground. The distance traveled $s = H/2$, and the initial velocity $v_0 = 0$, so

$$-\frac{H}{2} = 0t_1 - \frac{1}{4}(1 - \frac{1}{\sqrt{2}})gt_1^2,$$

which yields $t_1 = \sqrt{\dfrac{2H}{(1 - \frac{1}{\sqrt{2}})g}}$.

(c)† Using the kinematic formula,

$$v_f^2 = v_0^2 + 2as\,,$$

we find the speed v_f of M_2 (and of M_1) when M_2 hits the ground,

$$v_f = \sqrt{\frac{gH}{2}(1 - \frac{1}{\sqrt{2}})}\,.$$

† (c) *can also be answered without calculation: In order for a mass to strike the pulley it must be raised vertically a distance H/2, while the other equal mass is lowered vertically an equal distance H/2 – the potential energy gained by one equals that lost by the other, so no energy at all can be lost in this process. However, in fact, some energy is irretrievably lost (to vibration, heat, etc.) when the downward-accelerating mass hits the ground; after that there must be insufficient energy left in the system to raise the upward-moving mass as high as the downward-moving mass fell, so the former can not strike the pulley.* Mike Gottlieb

Now we use the same kinematic formula again, with $v_0 = v_f$ and $a = \frac{Mg}{\sqrt{2}}$, solving for s to find how much further M_1 travels along the inclined plane after M_2 hits the ground:

$$s = \frac{H}{4}(\sqrt{2} - 1).$$

The total distance traveled by M_1 along the inclined planes is thus,

$$H/2 + s = H(\frac{1}{2} + \frac{1}{4}(\sqrt{2} - 1)).$$

On the other hand, the length of the rope from the pulley to M_1 is initially $\frac{H}{\sqrt{2}}$.

Therefore, since $H(\frac{1}{2} + \frac{1}{4}(\sqrt{2} - 1)) < \frac{H}{\sqrt{2}}$ $\left[(0.604)H < (0.707)H\right]$ M_1 will stop before it hits the pulley.