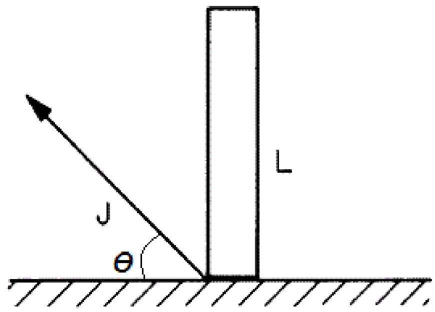

Figure 1:

1 Impelled rod

An upright rod of mass M and length L is given an impulse J at its base, directed at ϑ upward from the horizontal, which sends the rod flying. What value(s) should J have so that the rod lands vertically again (i.e., upright on the end at which J was applied)?

1.0.1 Solution

The *Impulse-Momentum Theorem* and the *Angular Impulse Momentum Theorem* will be used.

Impulse Momentum Theorem applied to the J**_vertical component** $(J\sin(\vartheta) = F_z dt = d(MV_G) = MV_G^0 - 0)$:

$$J\sin(\vartheta) = MV_G^0 \Longrightarrow V_G^0 = \frac{J\sin(\vartheta)}{M}$$

where V_G^0 is the vertical initial velocity of the center mass G.

It follows that at time t:

$$V_G(t) = V_G^0 - gt$$

where g is the gravity acceleration. $V_G(t)$ will be zero when $t = V_G^0/g$. It follows that G will be again at height $L/2$ after the time:

$$T_G = 2\frac{V_G^0}{g} = 2\frac{J\sin(\vartheta)}{gM} \tag{1}$$

Angular Impulse Momentum Theorem referred to the center of mass G:
$(-J\cos(\vartheta)L/2 = F_x L/2dt = d(I\omega) \simeq I\omega - 0)$ where $I = ML^2/12$ is the rod's moment of inertia and ω the initial angular velocity. It follows:

$$I\omega = -J\cos(\vartheta)L/2 \Longrightarrow \omega = -J\cos(\vartheta)L/(ML^2 2/12) = -6J\cos(\vartheta)/(ML)$$

As a consequence a rotation of magnitude 2π will occur in a time:

$$T_{2\pi} = 2\pi/|\omega| = \pi ML/(3J\cos(\vartheta)) \tag{2}$$

For the rod to land vertically on its base it must be: $T_G = nT_{2\pi}$ $(n = 1, 2, 3, ...)$ and then from (1) and from (2) it follows:

$$2\frac{J\sin(\vartheta)}{gM} = n\pi ML/(3J\cos(\vartheta))$$

This equation $\forall\vartheta, 0 < \vartheta < \pi/2$ admits the solution:

$$J = M\sqrt{\frac{n\pi gL}{6\sin(\vartheta)\cos(\vartheta)}} = M\sqrt{\frac{n\pi gL}{3\sin(2\vartheta)}}$$