Bowling Ball Rolling

A uniform bowling ball of radius $R$ and mass $M$ is initially launched so that it is sliding with speed $V_0$ without rolling on an alley with a coefficient of friction $\mu$. How far does the ball go before it starts rolling without slipping, and what is then its speed?

**Sukumar Chandra’s Solution (using kinematics)**

![Diagram of bowling ball](image)

When the ball is slipping, the forces acting on it are:

1) Weight $Mg$, acting vertically downward through the centre,
2) Normal reaction $N$, vertically upward through the point of contact
3) Forces of kinetic friction, $f_k = \mu N$, horizontally through the point of contact and opposite to the direction of motion.

The only horizontal force $f_k$ produces horizontal acceleration, $a = \frac{\text{force}}{\text{mass}} = \frac{\mu N}{M} = \mu g$ [as $N = Mg$]. This acceleration is directed opposite to the direction of motion of the ball and hence its velocity $V$ at any instant $t$ is given by:

$$V = V_0 - \mu gt. \quad (1)$$

Further, the only force that produces a torque about the centre is $f_k$. This torque is of magnitude $f_kR$, acting in anticlockwise direction producing an anticlockwise angular acceleration, $\alpha$, of the ball about its center given as $f_kR = I_{cm} \alpha$, or $\mu MgR = \frac{2}{5}MR^2 \alpha$, or $\alpha = \frac{5 \mu g}{2R}$. This angular acceleration sets the ball rotating with increasing angular velocity in anticlockwise direction whose magnitude $\omega$, at any instant $t$, is given by

$$\omega = \alpha t. \quad (2)$$

When the ball starts pure rolling,

$$V = \omega R \quad (3)$$

is satisfied. Hence using equations (1), (2) and (3) we get, $V_0 - \mu gt = \alpha tR = \frac{5 \mu gt}{2}$, or $\mu gt = 2V_0 / 7$. Putting this value back in equation (1), we get $V = 5V_0 / 7$.

Again, using the equation of kinematics, $v^2 = u^2 + 2aD$, we get $(5V_0 / 7)^2 = V_0^2 - 2\mu gD$ or, $D = 12V_0^2 / 49\mu g$. 