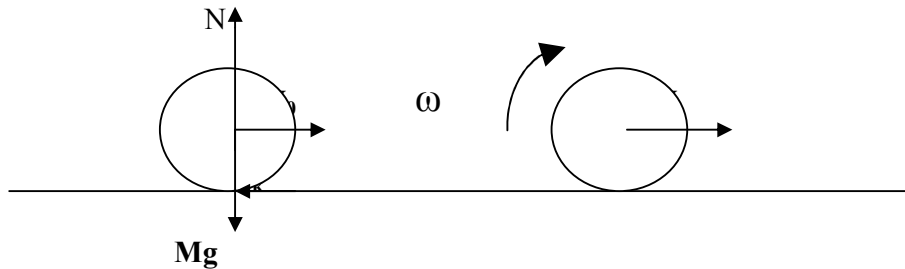


## Bowling Ball Rolling

A uniform bowling ball of radius  $R$  and mass  $M$  is initially launched so that it is sliding with speed  $V_0$  without rolling on an alley with a coefficient of friction  $\mu$ . How far does the ball go before it starts rolling without slipping, and what is then its speed?

### Sukumar Chandra's Solution (using conservation of angular momentum)



When the ball is slipping, the forces acting on it are:

- 1) Weight  $Mg$ , acting vertically downward through the centre,
- 2) Normal reaction  $N$ , vertically upward through the point of contact
- 3) Forces of kinetic friction,  $f_k = \mu N$ , horizontally through the point of contact and opposite to the direction of motion.

Vertical equilibrium suggest  $N = Mg$ . The only unbalanced force  $f_k$  acts horizontally through point of contact and hence external torque about any axis of rotation passing through the floor and normal to the plain of motion is zero. So angular momentum of the ball about this axis is conserved. Its initial angular momentum about this axis is  $MV_0R$ . When it starts pure rolling with translational speed  $V$  and angular speed  $\omega$ , its angular momentum about this axis is  $MVR + I_{cm}\omega$ , where  $I_{cm} = \frac{2MR^2}{5}$  is the moment of inertia of the ball about it centre of mass. As it is pure rolling now so  $\omega = V/R$ . Hence conservation of angular momentum implies:  $MV_0R = MVR + I_{cm}\omega$ , or  $MV_0R = 7MVR/5$ , or  $V = 5V_0/7$ .

Translational acceleration of the ball,  $a = \text{force/mass} = -\mu Mg/M = -\mu g$ . Using the equation of kinematics,  $v^2 = u^2 + 2aD$ , we get  $(5V_0/7)^2 = V_0^2 - 2\mu gD$  or,  $D = 12V_0^2/49\mu g$ .