A small solid rubber ball of radius \( r \) is thrown against a rough horizontal floor such that its velocity just before striking the floor at \( A \) is \( v \) making an angle of \( 60^\circ \) with the horizontal and also has a back spin of angular velocity \( \omega \). It is observed that the ball bounces from \( A \) to \( B \), then from \( B \) to \( A \), then from \( A \) to \( B \), etc. Assuming perfectly elastic impact determine

a) the required magnitude of \( \omega \) of the back spin in terms of \( v \) and \( r \).

b) the minimum magnitude of co-efficient of static friction, \( \mu_{s\text{ min}} \), to enable this motion.

**Sukumar Chandra’s Solution**

At \( A \), the ball strikes the floor with linear velocity whose horizontal component is leftward \( v \cos 60^\circ \) or \( v/2 \) and vertical component is downward \( v \sin 60^\circ \) or \( \sqrt{3}v/2 \). Its angular velocity is clockwise \( \omega \). In order that the ball executes the repeated to and fro motion it must bounce back with linear velocity whose horizontal component is rightward \( v/2 \), vertical component is upward \( \sqrt{3}v/2 \) and angular velocity anti-clockwise \( \omega \). Hence during the small duration \( \Delta t \) the ball, of mass \( m \) (say), is in contact with floor, its

1) change in linear horizontal momentum is rightward \( mv \). Hence, \( f_s \Delta t = mv \), where \( f_s \) is the average static impulsive force of friction that comes in to play horizontally rightward during contact.

2) change in linear vertical momentum is upward \( \sqrt{3}mv \). Hence, \( N \Delta t = \sqrt{3}mv \), where \( N \) is the average impulsive normal reaction force that comes into play during the contact.

3) change in angular momentum is \( 2I\omega \) in anti-clockwise direction, where \( I \) (= \( 2mr^2/5 \)) is the moment of inertia of the solid spherical ball about its axis of rotation through its center.

a) As torque impulse = change in angular momentum, so \( f_s \Delta t \times r = 2I\omega \), i.e. \( mv = 2(2mr^2/5) \omega \),

Hence \( \omega = 5v/4r \).

b) Also the limiting value of static force of friction is \( \mu_s \) times the normal reaction force, i.e. \( f_s \leq \mu_s N \)

\[ \Rightarrow f_s \Delta t \leq \mu_s N \Delta t \]

\[ \Rightarrow mv \leq \mu_s \sqrt{3}mv \]

\[ \Rightarrow \mu_s \geq 1/\sqrt{3} \]

\[ \Rightarrow \mu_{s\text{ min}} = 1/\sqrt{3} \approx 0.58. \]