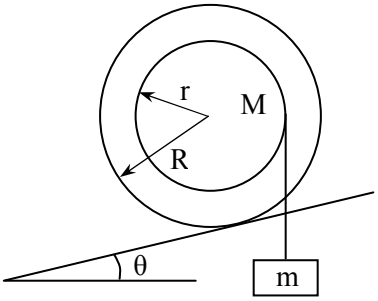
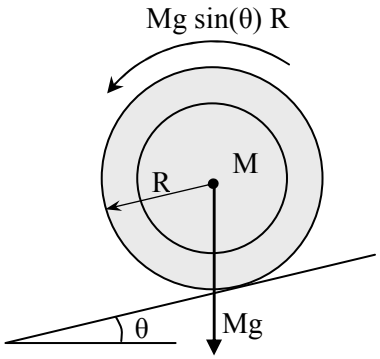
bobbin on incline



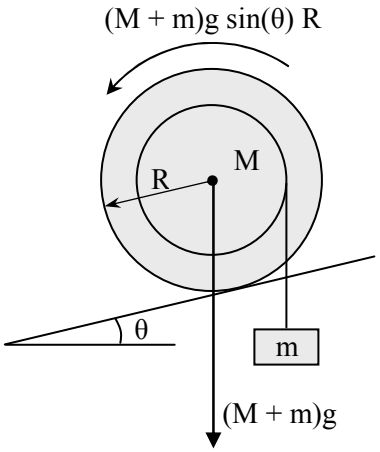
A bobbin of M = 3 kg consists of a central cylinder of radius r = 5 cm and two end plates of radius R = 6 cm. It is placed on a slotted incline on which it will roll but not slip, and a mass m = 4.5 kg is suspended from a cord wound around the bobbin. It is observed that the system is in static equilibrium. What is the angle of tilt θ of the incline?

Solution by Thomas Knapp:

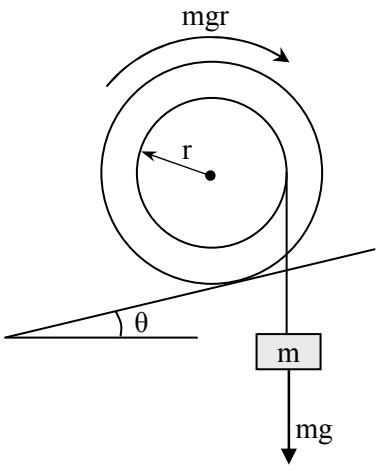
In the absence of the suspended mass, the weight of the bobbin would exert a torque that would tend to make the bobbin roll down the incline. The magnitude of this torque would be equal to the component of the bobbin's weight parallel to the incline times the radius of the bobbin's end plates:



However, the additional weight of the suspended mass increases this torque:



The weight of the suspended mass also exerts a torque on the bobbin in the opposite direction. The magnitude of this torque is equal to the weight of the suspended mass times the radius of the bobbin's central cylinder:



Since the bobbin is in static equilibrium the magnitudes of these opposing torques must be equal,

$$(M+m)g\sin\theta\, R = mgr\,.$$

So,

$$\sin\theta = \frac{mr}{(M+m)R} = \frac{4.5\cdot 5}{6(3+4.5)} = 0.5\,.$$

Therefore $\theta = 30°$.