Ball Up/Down

If you throw a small ball vertically *up*ward in real air with drag, does it take longer to go *up* or come down?

Solution by Sukumar Chandra

While the ball is going *up*ward the forces of gravity and air drag are both downward, hence it decelerates with an average magnitude $a_{up} > g$ (the acceleration due to gravity).

If the ball is thrown with speed v and it takes time t_{up} to reach the highest point h where its velocity is zero, then using the equation of kinematics we can write

$$v^2 - 2\, a_{up}\, h = 0,$$

and

$$v - a_{up}\, t_{up} = 0.$$

Eliminating v we get

$$t_{up} = \sqrt{\frac{2h}{a_{up}}}\,.$$

While coming down, the force of air drag is *up*ward and the force of gravity is downward, so the ball's downward acceleration has average magnitude less than g, say $a_{down} < g$.

If the ball takes t_{down} seconds to come down to Earth starting from rest and covering the height h,

$$h = \frac{1}{2} a_{down} t_{down}^2\,,$$

or

$$t_{down} = \sqrt{\frac{2h}{a_{down}}}\,.$$

Hence we have

$$\frac{t_{up}}{t_{down}} = \sqrt{\frac{a_{down}}{a_{up}}}\,.$$

As $a_{up} > a_{down}$, so $t_{down} > t_{up}$, thus it takes longer coming down.