8/26 18:02

Here's my guess: a simple pendulum with period 1.00s has a length of about 0.249m. The pendulum in question is being made to behave like a pendulum with a period of 1.10s which has a length of about 0.301m; and who's amplitude at a point 0.052m down from the pivot is 1.00cm. The amplitude at the bottom should be approximately (0.301m / 0.052m) * 1.00cm, or 5.8cm.

This person seems to have the right idea, in terms of the geometry of the problem. However, all statements in a solution must be justified, and the first two statements, giving the lengths of simple pendulums with periods 1.00s and 1.10s, aren't justified in any way, which disqualifies this solution. Furthermore, these statements appear to be applications of the exact formula $L = (g/4\pi^2)T^2$ which is the solution of a differential equation, the use of which is forbidden without other justification not involving calculus, etc.

8/27 10:51

That is a nice problem. Hope you don't mind me taking a shot.

First, I'm assuming a linear oscillator. The envelope of bob's oscillation must have a period of 11 seconds, since that's the time it takes for the relative phase between two oscillation to go around. The envelope is sinusoidal, and must pass through the node at the same slope as the envelope of a resonance, since it's effectively at resonance at these points. (The phase of the support's displacement leads or lags bob's displacement by exactly 90° there.)

So the only question is at what rate would amplitude of bob's oscillation grow if both periods were at 1s exactly. The kinetic energy of the bob as it passes through the node is $(1/2)mv^2$. The force due to bob displacement is F=kx for some k and x=1cm. We don't know what the k or m is, but $k/m=\omega^2$, and that's available. Rate of change of pendulum's energy is therefore F*v. With total energy related to current amplitude by $E=(1/2)ka^2$, where a is amplitude. $v=a/\omega$. Energy increases at the rate of kxa/ω. So a^2 is increasing at rate of $2xa/\omega$. Since a(t)=ct (resonance), $a^2=c^2t^2$ which is area under a triangle $y=2c^2t$, and therefore increases at a rate of $2c^2t = 2ac$. So the rate of increase of $a=x/\omega=x/2\pi$.

So the solution is the amplitude of the sinusoidal wave with period of 11 seconds and passes through zero at slope of $x/2\pi$. Sin with period 2π will pass through zero at slope 1. So $A=(x/2\pi)*(11/2\pi) =11*x = 11$cm.

(Later clarified: A = a/(1-P/p) = 1 cm / (1 – 1/1.1) = 11 cm)

Wrong answer. Wrong solution too. (Note: steady-state was not assumed.)

8/28 21:16

(Commenting on the above)

That is not the correct answer. The correct answer should read (for small oscillations of the pendulum, small meaning the amplitude of oscillaiton measured by the maxium angle of deflection from the vertical is much smaller than 1 rad):

A = a/[(T_{pivot}/T_{pend})^2 - 1] = 1 cm / [(1.1/1)^2-1] = 4.8 cm

This is not a solution, and the given answer is wrong. (The answer is correct in the non-inertial reference frame of the moving pivot.)

8/29 13:37

Now, I am going to show you jokers how we solve this problem with differential equations at Caltech. After that, everyone will know the right answer (which is not 11 cm and not 4.8 cm), and how to find it in a conventional way. The challenge to find it in an unconventional way remains. However, [name deleted] and [name deleted] are disqualified from the competition (for having submitted wrong answers). Please excuse me for typing this out in ASCII - I don't have time to make it pretty.

The equation of motion for a forced harmonic oscillator with one degree of freedom is

mx" + kx = F(t),

or

(1) x" + (k/m)x = F(t)/m,

where F(t) is the driving force. (Note: in this problem x is the horizontal position of the bob.) The frequency of _unforced_ oscillations, whose period is given to be T0 = 1 sec, is

w0 = Sqrt(k/m) = 2Pi/T0 = 2Pi sec^-1,

while the frequency of _forced_ oscillations, whose period is given to be T = 1.1 sec is

w = 2Pi/T = 2Pi/1.1 sec^-1.

Let X be the position of the pivot, such that

X = X0 cos(wt), with X0 = 1 cm.

Then we must have (by Newton's law),

mx" + k(x-X) = 0,

and thus,

(2) x" + (k/m)x = (k/m)X0 cos(wt) = w0^2 X0 cos(wt) .

Comparing (2) to (1), we find that the driving force is

F(t) = F0 cos (wt), with F0 = kX0,

thus (1) becomes,

(3) x" + (w0^2)x = (w0^2)X0 cos(wt).

When steady-state motion is attained we have

(4) x = A cos(wt),

where A is the amplitude we are seeking. Substituting (4) into (3), dividing both sides by the common factor cos(wt), we get

-Aw^2 + Aw0^2 = X0(w0^2),

and thus,

A = X0 (w0^2)/(w0^2 - w^2)

= X0 1/(1-(w/w0)^2)

= 1 cm * 1/(1-(1/1.1)^2)

~= 5.76 cm.

This is not a contest entry, but my own post on the Physics Forum, which I include for the sake of completeness – noting that after this posting the answer to the problem became public knowledge.

8/29 15:42

I shall name the natural period P0 = 1.00 second, amplitude A = 1.00 cm and period P = 1.10 seconds

Since the natural amplitude is 1.00 seconds, I tried to equate the A = 1.00 cm and P = 1.10 seconds to a numerical answer of 1 cm.

Since (2pi/1.1 seconds) is equal to 5.71 Hz, The pendulum needs to travel an equal distance in amplitude measure to create a steady motion multiplied by time. (d =

v*t). (5.71 cm / 5.71 Hz) = (1 cm*sec/P0) = 1cm

So this concludes an amplitude of 5.71 cm +/- 0.03 cm will result in a steady motion.

As Pauli might say, this solution is "not even wrong!"

9/20 13:40

The answer for this exercise challange is 1.21 cm

Not a solution, and wrong answer.

10/18 8:44

Ok, here goes. To be honest, this feels hopelessly naive, but I might as well get the discussion going...

In the steady state, the bob's swing must clearly take 1.1s.

As a rough approximation, one can imagine this swing looking like the natural swing, but with a flattish portion in the centre, with a duration of 0.1s. (In reality the swing would resemble a distorted figure of 8, but for the purposes of this exercise I think the approximation is ok.)

Aside: To get an idea of the effect of the forcing motion, let's consider the two extremes:

1. Very slow oscillation: In this case, the motion of the bob would almost exactly match the motion of the pivot point. The 'flat' portion of the swing would take the entire duration of the swing, and the amplitude of the bob would be 1cm.

2. Oscillation = 1s: In this case, the motion of the pendulum is continually reinforced by the pivot point motion, and the amplitude of oscillation increases to the maximum (or rather, until the pendulum stops being a pendulum and hits someone in the face). The 'flat' portion of the swing becomes neglible.

So this looks like we have some sort of reciprocal relationship going on, perhaps:
amplitude = 1cm + (1 / n (period - 1))^m

Anyway, according to our dodgy approximation above, the flat portion must take 0.1s, and the whole swing takes 1.1s, that leaves us with 1s for the 'natural', roughly sinusoidal part of the swing.

The motion of the pivot adds 1cm to the natural swing, and this distance must be covered in the extra 0.1s we have available. Assuming that this is largely done at the bottom of the swing, then we can say that the maximum velocity is 10cm/s.*

For the pendulum to reach this speed in 0.5s, it has been accelerating at ~20cm/s^2, and the distance it has covered must be 20 x 0.5 x 0.5 / 2 = 20 / 8. (Do I need to derive s = 1/2at^2?)

This is half the amplitude of the normal swing, so the total amplitude, including our 1cm extra, is:

1 + (2 x 20 / 8) = **6cm**

Which implies that the relationship could be:

amplitude = 1cm + (1 / 2 x (period - 1))

Couple of dodgy approximations in here, so I'm sure someone can do better!

(Disclaimer: I've never seen this problem before, and to my shame I haven't even read the lectures yet...)

*This is the part I am most unsure of - it only really works if the bob swing is very large compared to the motion of the pivot. In reality, this sets the upper limit to the bob speed, and therefore the actual swing will be smaller.

Another solution in the "not even wrong" category.

10/23 16:41

Feynman's lectures abound in beautiful examples about how to approach complicated physical situations. Before writing equations, a good idea is to start looking for related systems with well known behavior, and estimate to what degree their behavior matches the answer to our problem. In most cases, the answer is not exact, but at least will give a hint about the way forward for solving the initial problem.

Applied to our case, such a procedure leads to an almost geometrical and "visual" solution. Instead of directly solving the proposed problem, we look at the situation shown in Fig.1(a), where the pendulum with a fixed pivot point $P^{/}$ and length L experiences small oscillations of amplitude $L\theta \cong A_b$ and period $T = 2\pi\sqrt{L/g}$ (g stands for the acceleration of gravity).

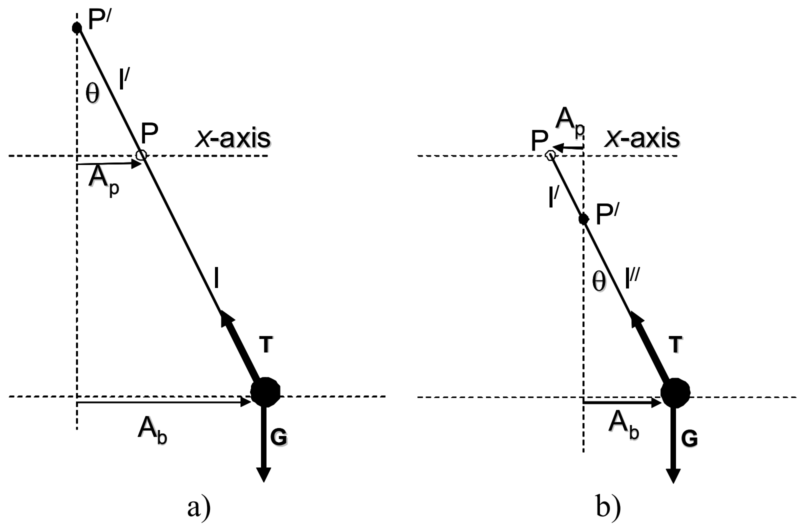

FIG. 1: (a) A simple pendulum with a fixed pivot point P/ and its length $L = l + l'$ adjusted to provide a natural period $T > T_0$ ($T_0 = 2\pi\sqrt{l/g}$). (b) The fixed pivot point P/ satisfies $l = l' + l''$ and $T < T_0$.

Up to the multiplicative constant 2π , the latter equation can be easily derived using dimensional analysis, a fact which is largely known. The constant itself plays no significant role in the solution below. The intersection P of the pendulum rod with the horizontal *x*-axis will obviously oscillate in phase and at the same frequency with some amplitude A_P which we can estimate in an elementary way. Now, assume that, for some reason, the region above *x*-axis is invisible to us. *All we can see is a steady motion where our simple pendulum of length l (now with small variations in time) has the pivot point P moved laterally in a practically sinusoidal motion* with amplitude A_b and the angular frequency $\omega = 2\pi / T$ given by

$$\omega^2 = g/(l + l') = \omega_0^2 / [1 + (l'/l)], \tag{1}$$

where $\omega_0 = l / g > \omega$. Despite this is a slightly different situation since the length l shows small variations, and the pivot point do not move exactly along *x*-axis, as long as the friction is negligible and the amplitude of oscillations remains small, we can safely disregard such details and use the model as a good approximation of our initial system. Now, all we have to do is to use the theorem about similar triangles to obtain the ratio of the two amplitudes in Fig. 1(a)

$$A_b / A_P = (l + l')/l' = 1 + (l/l'). \tag{2}$$

Inserting the ratio l/l' from (1) we can directly solve for the amplitude of the pendulum blob

$$A_b = A_P / \left[1 - (\omega/\omega_0)^2\right] = A_P / \left[1 - (T_0 / T)^2\right]. \tag{3}$$

Using the provided numerical data we get $A_b \cong 5.8$ cm. The blob amplitude <u>*computed with respect to P*</u> is therefore given by the difference $A_b - A_P = A_P / \left[(T/T_0)^2 - 1\right] \cong 4.8$ cm.

Using similar approximations we can also tackle the case $T < T_0$ (FIG.1,b), $\omega^2 = g / l'' = \omega_0^2 / [1 - (l'/l)]$, and $A_b / A_p = (l/l') - 1$ (using similar triangles in FIG. 1,b). A straightforward calculation yields to

$$A_b = A_p / \left[(\omega/\omega_0)^2 - 1\right] = A_p / \left[(T_0/T)^2 - 1\right]. \quad (4)$$

This time, the blob amplitude *computed with respect to P* is given by the sum $A_b + A_P = A_P / \left[1 - (T/T_0)^2\right]$. As we can see, eqs. (3) and (4) provide us with a general solution valid for small oscillations and very low friction

Amplitude with respect to the mobile pivot $= \pm A_P / \left[(T/T_0)^2 - 1\right]$, (5)

with the plus sign used when $T > T_0$ (below resonance), and the minus sign when $T < T_0$ (above resonance). When using $T = 0.91\,\text{s}$, we obtain $A_b \cong 4.8\,\text{cm}$, and the blob amplitude *computed with respect to P* is given by $A_b + A_P = A_P / \left[1 - (T/T_0)^2\right] \cong 5.8\,\text{cm}$. This is quite enlightening because without any fancy calculation we were able to estimate the magnitudes and to show how the blob amplitudes behave with those oscillations of P around resonance. Moreover, under natural assumptions (steady motion, very low friction, and small amplitudes) the model above shows that below resonance ($\omega = \omega_0 - \Delta\omega$) the blob oscillates *in phase* with the oscillating pivot P (FIG. 1,a), while above the resonance ($\omega = \omega_0 + \Delta\omega$) the pendulum blob oscillates in *phase opposition* with the oscillating pivot P (FIG. 1,b). Therefore, when traversing a resonance the pendulum blob experiences a change of π in its phase. Of course, we can confirm all these conclusions by conducting a full mathematical description of this system.

As a final step, we need to check that our estimated amplitudes satisfy the small angle approximation. The numerical values ($T_0 = 1\,\text{s}, g \cong 9.8\,\text{ms}^{-2}$) give for the length of our pendulum a value of $l \cong 25\,\text{cm}$. Therefore, the estimated angular amplitude is close to $\theta_{\max} \cong 0.2\,\text{rad}$, or $\theta_{\max} \cong 11^o$, which lies in the range of the desired approximation.

*This is a very well-written solution, however the authors states that "$T = 2\pi\sqrt{L/g}$," without justifying it, and that is the soluton to a differential equation,. Granted he mentions the fact that $T \propto \sqrt{L/g}$ can be derived by dimensional analysis (without actually doing so), but then, curiously, he writes "*The constant [2π] itself plays no significant role in the solution below*," when, in fact, it plays no role whatsoever . Eq. (1) is the solution to a differential equation, which disqualifies this solution. I will remark, however that it was unnecessary for the author to write Eq. (1), since his solution does not require the exact (given) relationship between ω, g and L (nor between ω_0, g and l), but only the ratio l/l', and $l/l' = l/(L-l) = 1/(L/l - 1) = 1/\left((T/T_0)^2 - 1\right)$, using the result of the dimensional analysis.*

10/24 18:06

First of all let's imagine that you do not move the pendulum by hand but it is suspended from the top of a car wich moves in a sinusoidal way. So if the car has an aceleration a then the bob will fill a force wich is F=-m*a where m is it's mass. We have to include that force because the system is not inertial. Now we can just write Newton's law. Since it is acelerating in the x direction then this is also the direction of the force F. The gravity force mg is in the y direction always. The other force that apears is the tention T. By analyzing the forces into a component paralel and a perpendicular to the force we find that $T=m*g*\cos\theta+F*\sin\theta$. The total force in the x direction is $T*\sin\theta-F$. (θ is the angle of the pendulum with respect to the y axis) So now the total force in the x direction is given by $\Sigma F =- F +m*g*\cos\theta * \sin\theta + F*\sin\theta*\sin\theta$.

Here we point out that we want to study only small oscilations, when θ is really small. So we make the aproximation $\sin\theta=\theta$ and $\cos\theta=1$. You can now see why we study only the motion of the x axis. If L is the length of the pendulum's "rope" then $x=L*\sin\theta=L*\theta$ and $y=L*\cos\theta=L$. So we only need to study the projection of the motion of the x axis. You can imagine that as studying the shadow of the bob on the floor created by some light coming just over the bob in the y axis.

The net force in the x direction in first order of θ is $\Sigma F = -F + m*g*\theta$. But $\theta=x/L$, so $\Sigma F=+m*a+m*x*g/L$. However g/L is the square of the pendulums natural frequency w. Since our car is moving in a sinusoidal motion, let it be $B*\cos(\omega t)$, we know from Hook's law that the aceleration will be $a=-\omega^2 *B*\cos(\omega t)$. What will the form of x be? Well there is something special about sines and cosines. They are only "compatible" with their selfs. A sine or a cosine of some frequency never contains other frequences and a motion like that can only be steady if it is the compatible one. (Remember for example that the only way for the oscilation of a music cord to be steady is to be a characteristic of the cord.) The only way we can achieve a steady motion is for x to be proportional to $\cos(\omega t)$, so $x=A*\cos(\omega t)$. What the problem is asking is A.

Now the total force can be writen as
$\Sigma F=m*(-\omega^2 *B+m*A*w^2)*\cos(\omega t)$
This is the equation of motion for the x direction...or the shadow if you like it better.

Well there is however a nice coincidence. If you just saw the shadow of this object you could not decerne it from another object doing a really familiar motion. I am talking about a circular motion with constant speed $u=\omega*R$! The net force was calculated geometrically by Newton and it is
$m*\omega^2*R$, where R is the radius of motion. The force in the x direction however is $m*\omega^2*R*\cos(\omega t)$, exactly as the one in our problem.

Of course every force creates only one kind of motion. You just have to specify the distance and the velocity and the solution is unique. We already specified the velocity at t=0 when we chose the cosine as the sinusoidal movement. We now need to specify the other one. Well...we want the maximum displacement to be A. That means allow the particle in the second problem to reach up to A, which means that the radius R will be $R=A$.

A solution to our problem will be that the forces in our two problems are equal.
$m*(-\omega^2 *B+m*A*w^2)*\cos(\omega t) = m*\omega^2*A*\cos(\omega t)$
which leads to
$A=B*\omega^2/(w^2-\omega^2)$
plugging in
$B=1.00$ cm

ω=1.10 s
w=1.00 s
we get

A= 5.76 cm

*This solution, besides being very convoluted and hard to read, makes contradictory statements about " ΣF." It furthermore states that "*g/L is the square of the pendulums natural frequency w,*" which is the exact solution of a differential equation. So, it is disqualified*

11/1 13:31

We know that a certain force is applied on the bob when the pivot point is moved. If we switch reference frames and move with the pivot we can pretty much see that the force applied is (roughly, I suppose, assuming the bob does not begin to move before we get to the first maximum amplitude) the same as the one required to move the bob to an amplitude of 1cm (neglecting the mass of the pivot since it's not so important). With this in mind we know the maximum force from the mass of the bob and the force equation of simple harmonic motion:

$$F_{original} = m_{bob}\omega_{pivot}^2 A_{pivot}$$

We also know that we can define natural angular velocities for the bob and pivot from their periods; we're going to need them soon:

$$\omega = \frac{2\pi}{T}, \qquad \omega_{pivot} = 5.7199, \qquad \omega_{bob} = 2\pi$$

At steady state when the bob is at its maximum amplitude, we can change reference frames again (do it twice now) to determine the force on the bob due to the pivot and vice versa. There will be a difference as effectively we are assuming different pendulum lengths (the different angular velocities) in each frame. The difference in the forces we call the net force on the bob:

$$F_{net} = m_{bob}(\omega_{bob}^2 - \omega_{pivot}^2)A_{bob}$$

Equating F_{net} and $F_{original}$ as they should be at least roughly the same at the maximum amplitude and cancelling the mass of the bob from the equations:

$$\Rightarrow A_{bob} = \frac{\omega_{pivot}^2 A_{pivot}}{\omega_{bob}^2 - \omega_{pivot}^2} = 4.762\ cm$$

Now I guess this is relative to the pivot since that's how we did the net force so we'll add one centimetre to make it absolute:

$A_{bob} = 5.762\ cm$

Disqualified because the first given equation is not justified but merely stated.

11/13 15:56

Resonant ratio of amplitudes goes like 1/(1-(f/f0)squared. If f is 0 ratio of amplitudes is 1; at low frequencies the pendulum follows the pivot. As f gets much greater than f0 the ratio goes to 0; at high frequencies the mass of the pendulum cannot accelerate to keep up with the pivot. If f equals f0 the ratio is undefined; with no damping the energy added to the system by moving the pivot keeps adding to the resonant mode of the pendulum. using this if the forcing period is 1.1 and the resonant period is 1.0 we get 5.76 cm.

Disqualified because the first given equation is not justified but merely stated.

11/14 4:18

The first step in the solution process is to obtain the equation of motion. To this end, I introduce a few symbols to represent physical quantities: m, l, g, being the mass of the pendulum, the length from the pendulum pivot to the mass center, and the gravitational constant. Without loss of generality, I assume a simple point mass pendulum with a massless rod, and no dissipation of any kind.

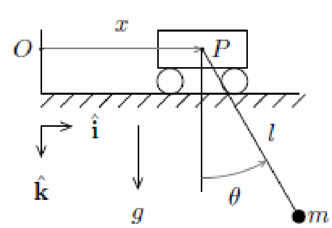

Figure 1: Pendulum with horizontally translating pivot. $\hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{i}}$ points out of the page.

The velocity and acceleration of the pendulum mass relative to an inertial frame are

$$\begin{aligned}\mathbf{v} &= \dot{x}\hat{\mathbf{i}} + \dot{\theta}\hat{\mathbf{j}} \times \left(l\sin\theta\hat{\mathbf{i}} + l\cos\theta\hat{\mathbf{k}}\right) \\ &= \left(\dot{x} + l\dot{\theta}\cos\theta\right)\hat{\mathbf{i}} + \left(-l\dot{\theta}\sin\theta\right)\hat{\mathbf{k}} \\ \mathbf{a} &= (\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)\hat{\mathbf{i}} + (-l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta)\hat{\mathbf{k}}\end{aligned}$$

Two forces act on the mass; the gravitational force $\mathbf{F}_g$ and the force of the rod $\mathbf{F}_r$:

$$\mathbf{F}_g = mg\hat{\mathbf{k}} \qquad \mathbf{F}_r = -F\sin\theta\hat{\mathbf{i}} - F\cos\theta\hat{\mathbf{k}}$$

where F is a function of time. Newton's 2nd law yields:

$$mg\hat{\mathbf{k}} - F\sin\theta\hat{\mathbf{i}} - F\cos\theta\hat{\mathbf{k}} = m(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)\hat{\mathbf{i}} + m(-l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta)\hat{\mathbf{k}}$$

Dotting this equation with $\cos\theta\hat{\mathbf{i}} - \sin\theta\hat{\mathbf{k}}$, dividing through by m and rearranging, yields:

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0.01\left(\frac{2\pi}{1.1}\right)^2 \sin\left(\frac{2\pi}{1.1}t\right)\cos\theta$$

Notice how dotting the Newton's equation into the direction perpendicular to the rod is a convenient way to eliminate the constraint force $\mathbf{F}_r$. Linearizing about the downwards position $\theta = 0$, we have $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, which yields the following second order linear differential equation:

$$\ddot{\theta} + \frac{g}{l}\theta = 0.01\left(\frac{2\pi}{1.1}\right)^2 \sin\left(\frac{2\pi}{1.1}t\right)$$

Defining $\omega_n^2 = \frac{g}{l}$ and $f_0 = 0.01\left(\frac{2\pi}{1.1}\right)^2$, and $\omega = \frac{2\pi}{1.1}$, we can rewrite the equation of motion as

$$\ddot{\theta} + \omega_n^2\theta = f_0\sin\omega t$$

Assuming a particular solution of the form $\theta_p(t) = \Theta\sin\omega t$, differentiating twice with respect to time, and substituting into the equation of motion yields:

$$(-\omega^2 + \omega_n^2)\Theta\sin(\omega t) = f_0\sin\omega t$$

which implies that the amplitude of swing of the pendulum bob after steady motion is attained is:

$$\begin{aligned}\Theta &= \frac{f_0}{\omega_n^2 - \omega^2} \\ &= \frac{0.01\left(\frac{2\pi}{1.1}\right)^2}{(2\pi)^2 - \left(\frac{2\pi}{1.1}\right)^2} \\ &= 0.047619 \quad \text{rad} \\ &= 2.7284 \quad \text{deg}\end{aligned}$$

Where we made use of the fact that the natural frequency is related to the natural period of oscillation by $\omega_n = 2\pi/T = 2\pi$ rad/s.

Note that this solution does not depend in any way on the physical parameters I introduced to show the derivation of the equation of motion, it only depends on the natural frequency of the pendulum, and the magnitude and frequency of the horizontal displacement. This solution is only valid for small angles, and assumes that friction is negligible.

Disqualified for using differential equations.

11/14 14:08

The energy transfer between the moving pivot and the pendulum stops, when the point corresponding to "would-be" pivot for T_forced=1.1second period of free oscillations finally stops moving. In other words, hang a 1.1-second-periodic free pendulum and "swing it" so that the point on its thread that corresponds to 1-second-period-pivot moves with amplitude 1cm. Done.

Let T_forced, T_free stand for 1.1-second pivot and 1-second pendulum periods, respectively. This point is (T_forced/T_free)^2=1.21 father from the weight of the pendulum than the actual pivot. Thus the amplitude

A_forced= A_pivot*(T_forced/T_free)^2/((T_forced/T_free)^2-1)=A_pivot/(1-(T_free/T_forced)^2)=5.76cm.

Sanety check: A_forced is much smaller than (g*(T_forced/(2pi))^2))=30cm - the pendulum "would-be" length as counted from the "would-be" pivot. Thus oscillations are small enough to be "independent enough" of the amplitude.

Finally, some hardly necessary hairsplitting. If we take into account pendulum nonlinearity, the answer would slightly increase, because the "free" period increases with amplitude, so that "the would-be pivot" does not need to move up as much. We can easily find the next iteration to A_forced as follows. The max angle of deviation is

Theta0=arcsin(A_forced/(g*(T_forced/(2pi))^2))=arcsin(5.76cm/30cm)=0.193.

The change in period is Theta0^2/16=0.0023. The answer would increase to A_pivot/(1-(T_free/T_forced)^2*(1+Theta0^2/8))=5.89cm - essentially the same.

Disqualified because the first given relation, between T_forced and T_free and the point corresponding to the"would-be" pivot, is not justified but merely stated.

11/14 15:12

I will refer to the 0.01m sinusoidal movement of the pivot point a driving force for the pendulum. There are two possibilities for the steady state behaviour for such a pendulum.

1. If the driving force has a longer period than the inherent period of the pendulum, the pendulum will move slower than usual, and in phase with the driving force, in such a way that the efective pendulum arm length is longer, and a stationary point of this imaginary longer pendulum arm will exist above the moving pivot point. Intuitively, this happens because the driving force is acting as a break on the natural pendulum movement, in order to slow it down.

2. If the driving force has a shorter period, the pendulum will move faster than usual, and out of phase with the driving force, creating an efectively shorter pendulum and a stationary point on the pendulum arm below the point where we apply the force. This happens because the driving force is dragging the inherent pendulum movement along to speed it up.

These approximations are valid as long as the driving force is only moving the pivot point a small distance compared to the original pendulum arm length, which in this case is easily calculated to be

$$L = g\left(\frac{T}{2\pi}\right)^2 \approx 0.25\text{m},$$

where we have inserted T =1s. We consider 0.01m « 0.25m to our required level of accuracy.

In this case, the driving force is slower than the pendulum, so the steady state is in phase, and the pendulum period must be the same as the driving force period, that is T =1.1s. From the above formula, we insert T =1.1s to get the efective pendulum arm length, which turns out to be

$$\bar{L} \approx 0.30\text{m}.$$

We know that the point at distance $\bar{L} - L \approx 0.05\text{m}$ downward from the stationary point along the imagined efective pendulum arm is moving 0.01m side to side. By scaling this side to side amplitude up to the full size of the efective pendulum, the pendulum is moving approximately an amplitude

$$A \approx 0.01\text{m} \times \frac{0.20\text{m}}{0.05\text{m}} = 0.06\text{m}$$

side to side.

Disqualified because the first calculation $L = g\left(\frac{T}{2\pi}\right)^2 \approx 0.25\text{m}$ *(and the next* $\bar{L} \approx 0.30\text{m}$*), is the solution to a differential equation (not otherwise justified).*

11/16 3:05

A pendulum with a period of 1.1 sec would have a length of .3m, while a pendulum with a period of 1 sec has a length of .248m. Imagine then that the pivot point of the real pendulum is a fixed point on an imaginary larger pendulum with length .3m. The real pivot point is attached to the imaginary one at a distance of .052m (the difference in lengths of the real and imaginary pendulums) from the pivot point of the imaginary pendulum. The pivot point of the imaginary pendulum is considered to be stationary. The real pivot point now moves with a sinusoidal motion, as specified in the problem. If we want the amplitude of the real pivot point to match that specified by the problem, .01m, then we need the angular amplitude of the imaginary pendulum's oscillation to be:

arcsin .01/.052 = 0.1935 radians

This will also then be the amplitude of the oscillation of the real pendulum.

The opening statement, giving the lengths of simple pendulums with periods 1.00s and 1.10s, uses T = 2 pi sqrt(L/g), the solution to a differential equation, which disqualifies this solution.

11/17 2:48

By definition the natural period is the period of the simple pendulum when the pivot is at rest. That, by dimensional analysis (using the fact that we have a unique way to get a time out of a simple pendulum dimensional quantities), gives the following relation between the period and the length of the simple pendulum:

T = c (l/g)^(1/2) = 1.0s where g is gravity, l is the length of the simple pendulum (that we don't know) and c is a constant we also don't know (at least not without using calculus or some experimental evidence).

Now we know from the problem that the system reaches a steady motion, and we argue that the steady motion, to be steady, has to be in phase with the sinusoidal motion of the pivot point that is forced to oscillate with period T'=1.1s, which means that when the oscillating pivot point is at its maximum right the pendulum is also at its maximum right (actually it could also be in counter phase if the pivot was moving with a period T' shorter than T). (the fact that being in phase or counter phase is a requirement of the steady motion is quite obvious, but just to make it explicit it's because by definition of a steady motion we don't want a motion with secular effects or quasi periodic effects).

Assuming that we are in the small angle approximation (which we can't check right away but we have to assume if we are using in any meaningful way the definition of natural period), we can see that a motion in phase with the pivot point looks like a longer simple pendulum of which we can just see the inferior part (this is not true in general, since the length of our pendulum doesn't change while oscillating, while in the longer pendulum I'm describing the length of the part below a certain line would actually be longer when the pendulum is at it's minimum height than when it is at its maximum height, so this is why we need the small angle approximation and we need to consider the length as being constant).

This means that our oscillating pivot point is just mimicking a "half-a-way" point of this longer simple pendulum (in the small angle approximation).

Now, because we want the pendulum to be in phase with the pivot point, this implies that the natural period of this longer pendulum is the same as the pivot point period, which is T' = 1.1s. But T', as we just explained, has to be also the period of the longer pendulum, which is, again by dimensional analysis:

$T' = c\,(l'/g)^{1/2}$ where l' is the length of the longer pendulum, and again, we don't know l' and c.

Now, because we are looking at the (longer) pendulum in the small angle approximation, we can look at two similar triangles defined by the (longer) pendulum itself.

One triangle is defined by the length of the longer pendulum and the distance from the vertical (with respect of the imaginary pivot point of the longer simple pendulum itself). The other triangle is defined by the distance of the horizontal oscillating pivot point and the imaginary fixed pivot point, and the amplitude of the oscillating pivot point (half of it).

Being the ratio of the two sides of these two triangles the same by euclidean geometry, we can write the following proportion:

$l'/(A'/2) = (l'-l)/(A/2)$ where A is the amplitude of the oscillating pivot point, $l'-l$ is the distance between the imaginary pivot point and the oscillating pivot point at it's maximum position, l' is the length of the imaginary longer pendulum, and A' is the amplitude we are supposed to calculate.

Now, from the expressions of the periods given above, we can just plug in and get an answer for A', which is

$A' = A\,(T')^2/[(T')^2-(T)^2] = (1\text{cm})\,(1.1\text{s})^2/[(1.1\text{s})^2-(1\text{s})^2] = 5.76\text{ cm}$

which is the answer we were looking for.

Now, three things are worth noting:

1) Even if used in the formulae, we never actually needed the values l, l', c, g, which would not change the answer as long as we stay in the small angle approximation;

2) from simple experimental observations, we do know what the constant c written above is, and that actually allows us to check that we really are in the small angle approximation for such periods and amplitudes (or lengths);

3) if the period of the forced oscillation of the pivot was shorter than the natural period of the simple pendulum, then the above procedure could still be applied to get the solution, as long as we consider the counter phase movement of the pendulum (which is, when the pivot is at its maximum right, the pendulum is at its maximum left). This, also in the small approximation, would create an imaginary pivot at a point lower than the horizontal oscillating pivot, allowing us to use different lengths for the period and find a shorter period.

This solution is correct.

11/26 9:25

Start with the following assumptions -

1. Assume the equilibrium solution for the x coordinate (small angle approximation) of the pendulum mass is of the form $A\sin(\omega t)$, where ω is the driven frequency.

2. Assume that when equilibrium is achieved that the driver no longer supplies any energy to the system.

Under these assumptions one can replace the pendulum with natural frequency ω_0 with length l_0 with one of length l that corresponds to natural frequency ω. The the problem of amplitude becomes one purely of geometry. First note that

$$l = l_0 \frac{\omega_0^2}{\omega^2}.$$

If $\omega < \omega_0$ then $l > l_0$ and the driver and pendulum motion are in phase, if $\omega > \omega_0$ then $l < l_0$ and the driven and pendulum motion are out of phase. Now consider figure 1 of an equivalent pendulum (below resonance). l is the length of the pendulum required for a natural resonance of ω. Then by geometry to get a driver amplitude of a we must have

$$\begin{aligned}
\frac{a}{l-l_0} &= \frac{A}{l} \\
A &= a\frac{l}{l-l_0} \\
A &= a\frac{l_0\frac{\omega_0^2}{\omega^2}}{l_0\frac{\omega_0^2}{\omega^2} - l_0} \\
A &= a\frac{\omega_0^2}{\omega_0^2-\omega^2}
\end{aligned}$$

or in terms of the period

$$\begin{aligned}
A &= \frac{a}{1-\left(\frac{\omega}{\omega_0}\right)^2} \\
&= \frac{a}{1-\left(\frac{T_0}{T}\right)^2} \\
&= \frac{1.0}{1-\left(\frac{1.0}{1.1}\right)^2}\ \text{cm} \\
&= 5.7619\ \text{cm}
\end{aligned}$$

which is the answer.

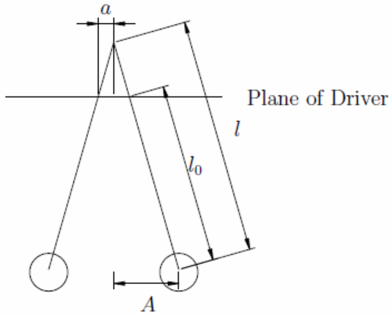

Figure 1: Equivalent Pendulum Below Resonance

Disqualified because the first statement $l = l_0\,\omega_0^2/\omega^2$ is not justified, but merely stated.

11/26 10:43

answer 1.21cm

Not a solution.

11/29 20:33

It was known by Galileo since the early 17th century (long before Newton's laws were expounded) that the period of a simple pendulum is proportional to the square root of its length. Or said another way, the length of a simple pendulum is proportional to its period squared.

$$l = kT^2$$

where k is a constant.

Imagine a longer simple pendulum P_1 of length l_1 with fixed pivot and natural period T_1=1.1 sec (identical to the excitation period of the pivot point of the given pendulum P_0 of length l_0 and natural period T_0 = 1 sec). Further imagine a point S on the longer pendulum P_1 a distance *d* from the pivot of P_1 (and distance l_0 from its bob) that undergoes oscillatory (harmonic) motion with amplitude *a* = 1 cm when the amplitude of the bob of P_1 is chosen appropriately. *The motion of pendulum P_0 in response to its harmonic pivot excitation hypothesized in the problem statement above is identical to the bottom portion of P_1 configured as a simple pendulum.*

Consider the (roughly) triangular region swept out by the bob of P_1 (and the bob of P_0) with base a_P we desire to calculate. Then by similar triangles

$$\frac{a_P}{l_1} = \frac{a}{d} = \frac{a}{l_1 - l_0}$$

Thus since $l_1 = kT_1^2$ and $l_0 = kT_0^2$ then

$$a_P = \frac{al_1}{l_1 - l_0} = a\frac{T_1^2}{T_1^2 - T_0^2} = a\frac{1.1^2}{1.1^2 - 1^2} = 5.76cm$$

Disqualified because the first statement, that "the length of a simple pendulum is proportional to its period squared" is not properly justified. [Historical "facts" don't count. For example, it was also "known" long before Galileo that an object needs a force applied to it to keep moving – that's what Aristotle said, and that's what people believed for a thousand years, but it was not true.]

12/13 3:11

as der will be an increased amplitude nd will be greater than the amplitude of the signal, as the energies of bob's motion will also add up, nd we know dat d energy is proportional to amplitude squared and v=w*amplitude for max velocity, so the change in amplitude will be a function of energy, now the increase in energy will cause the amplitude to increase, and more increase means more increase in amplitude, and of course it will be proportional to the energy, so the amplitude will be equal to $A = B |(w^2/w^2 - w'^2)|$, where b= amplitude of sine motion, w = frequency of period, w' = frequency of sine motion. thus we get after solving = 5.76 cm.

Judge for yourself.